

A brief history of the development of the Hoek-Brown failure criterion

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1980 Hoek E. and Brown E.T. 1980. *Underground Excavations in Rock*. London: Institution of Mining and Metallurgy 527 pages

Hoek, E. and Brown, E.T. 1980. Empirical strength criterion for rock masses. *J. Geotech. Engng Div., ASCE* **106**(GT9), 1013-1035.

The original failure criterion was developed during the preparation of the book *Underground Excavations in Rock*. The criterion was required in order to provide input information for the design of underground excavations. Since no suitable methods for estimating rock mass strength appeared to be available at that time, the efforts were focussed on developing a dimensionless equation that could be scaled in relation to geological information. The original Hoek-Brown equation was neither new nor unique – an identical equation had been used for describing the failure of concrete as early as 1936. The significant contribution that Hoek and Brown made was to link the equation to geological observations in the form of Bieniawski's Rock Mass Rating.

It was recognised very early in the development of the criterion that it would have no practical value unless the parameters could be estimated by simple geological observations in the field. The idea of developing a 'classification' for this specific purpose was discussed but, since Bieniawski's RMR had been published in 1974 and had gained popularity with the rock mechanics community, it was decided to use this as the basic vehicle for geological input.

The original criterion was conceived for use under the confined conditions surrounding underground excavations. The data upon which some of the original relationships had been based came from tests on rock mass samples from the Bougainville mine in Papua New Guinea. The rock mass here is very strong andesite (uniaxial compressive strength about 270 MPa) with numerous clean, rough, unfilled joints. One of the most important sets of data was from a series of triaxial tests carried out by Professor John Jaeger at the Australian National University in Canberra. These tests were on 150 mm diameter samples of heavily jointed andesite recovered by triple-tube diamond drilling from one of the exploration adits at Bougainville.

The original criterion, with its bias towards hard rock, was based upon the assumption that rock mass failure is controlled by translation and rotation of individual rock pieces, separated by numerous joint surfaces. Failure of the intact rock was assumed to play no significant role in the overall failure process and it was assumed that the joint pattern was 'chaotic' so that there are no preferred failure directions and the rock mass can be treated as isotropic.

1983 Hoek, E. 1983. Strength of jointed rock masses, 23rd. Rankine Lecture. *Géotechnique* **33**(3), 187-223.

One of the issues that had been troublesome throughout the development of the criterion has been the relationship between Hoek-Brown criterion, with the non-linear parameters m and s , and the Mohr-Coulomb criterion, with the parameters c and ϕ . Practically all software for soil and rock mechanics is written in terms of the Mohr-Coulomb criterion and it was necessary to define the relationship between m and s and c and ϕ in order to allow the criterion to be used to provide input for this software.

An exact theoretical solution to this problem was developed by Dr John. W. Bray at the Imperial College of Science and Technology and this solution was first published in the 1983 Rankine lecture. This publication also expanded on some of the concepts published by Hoek and Brown in 1980 and it represents the most comprehensive discussion on the original Hoek Brown criterion.

1988 Hoek E and Brown E.T. 1988. The Hoek-Brown failure criterion - a 1988 update. *Proc. 15th Canadian Rock Mech. Symp.* (ed. J.H. Curran), pp. 31-38. Toronto: Civil Engineering Dept., University of Toronto

By 1988 the criterion was being widely used for a variety of rock engineering problems, including slope stability analyses. As pointed out earlier, the criterion was originally developed for the confined conditions surrounding underground excavations and it was recognised that it gave optimistic results near surfaces in slopes. Consequently, in 1988, the idea of *undisturbed* and *disturbed* masses was introduced to provide a method for downgrading the properties for near surface rock masses.

This paper also defined a method of using Bieniawski's 1974 RMR classification for estimating the input parameters. In order to avoid double counting the effects of groundwater (an effective stress parameter in numerical analysis) and joint orientation (specific input for structural analysis), it was suggested that the rating for groundwater should always be set at 10 (completely dry) and the rating for joint orientation should always be set to zero (very favourable). Note that these ratings need to be adjusted in later versions of Bieniawski's RMR.

1990 Hoek, E. 1990. Estimating Mohr-Coulomb friction and cohesion values from the Hoek-Brown failure criterion. *Intl. J. Rock Mech. & Mining Sci. & Geomechanics Abstracts*. **12**(3), 227-229.

This technical note addressed the on-going debate on the relationship between the Hoek-Brown and the Mohr-Coulomb criterion. Three different practical situations were described and it was demonstrated how Bray's solution could be applied in each case.

1992 Hoek, E., Wood, D. and Shah, S. 1992. A modified Hoek-Brown criterion for jointed rock masses. *Proc. rock characterization, symp. Int. Soc. Rock Mech.: Eurock '92*, (J.Hudson ed.). 209-213.

The use of the Hoek-Brown criterion had now become wide-spread and, because of the lack of suitable alternatives, it was now being used on very poor quality rock masses. These rock masses differed significantly from the tightly interlocked hard rock mass model used in the development of the original criterion. In particular it was felt that the finite tensile strength predicted by the original Hoek Brown criterion was too optimistic and that it needed to be revised. Based upon work carried out by Dr Sandip Shah for his Ph.D thesis, a modified criterion was proposed. This criterion contained a new parameter a that provided the means for changing the curvature of the failure envelope, particularly in the very low normal stress range. Basically, the modified Hoek Brown criterion forces the failure envelope to produce zero tensile strength.

- 1994 Hoek, E. 1994. Strength of rock and rock masses, *ISRM News Journal*, 2(2), 4-16.
- 1995 Hoek, E., Kaiser, P.K. and Bawden, W.F. 1995. *Support of underground excavations in hard rock*. Rotterdam: Balkema

It soon became evident that the modified criterion was too conservative when used for better quality rock masses and a 'generalised' failure criterion was proposed in these two publications. This generalised criterion incorporated both the original and the modified criteria with a 'switch' at an RMR value of approximately 25. Hence, for excellent to fair quality rock masses, the original Hoek Brown criterion is used while, for poor and extremely poor rock masses, the modified criterion with zero tensile strength is used.

These papers (which are practically identical) also introduced the concept of the Geological Strength Index (GSI) as a replacement for Bieniawski's RMR. It had become increasingly obvious that Bieniawski's RMR is difficult to apply to very poor quality rock masses and also that the relationship between RMR and m and s is no longer linear in these very low ranges. It was also felt that a system based more heavily on fundamental geological observations and less on 'numbers' was needed.

The idea of *undisturbed* and *disturbed* rock masses was dropped and it was left to the user to decide which GSI value best described the various rock types exposed on a site. The original *disturbed* parameters were derived by simply reducing the strength by one row in the classification table. It was felt that this was too arbitrary and it was decided that it would be preferable to allow the user to decide what sort of disturbance is involved and to allow this user to make their own judgement on how much to reduce the GSI value to account for the strength loss.

- 1997 Hoek, E. and Brown, E.T. 1997. Practical estimates of rock mass strength. *Intl. J. Rock Mech. & Mining Sci. & Geomechanics Abstracts*. 34(8), 1165-1186.

This is the most comprehensive paper published to date and it incorporates all of the refinements described above. In addition, a method for estimating the equivalent Mohr Coulomb cohesion and friction angle was introduced. In this method the Hoek Brown criterion is used to generate a series of values relating axial strength to confining pressure (or shear strength to normal stress) and these are treated as the results of a hypothetical large scale in situ triaxial or shear test. A linear regression method is used to find the average slope and intercept and these are then transformed into a cohesive strength c and a friction angle ϕ .

The most important aspect of this curve fitting process is to decide upon the stress range over which the hypothetical in situ 'tests' should be carried out. This was determined experimentally by carrying out a large number of comparative theoretical studies in which the results of both surface and underground excavation stability analyses, using both the Hoek Brown and Mohr Coulomb parameters, were compared.

- 1998 Hoek, E., Marinos, P. and Benissi, M. (1998) Applicability of the Geological Strength Index (GSI) classification for very weak and sheared rock masses. The case of the Athens Schist Formation. *Bull. Engg. Geol. Env.* 57(2), 151-160.

This paper extends the range of the Geological Strength Index (GSI) down to 5 to include extremely poor quality schistose rock masses such as the 'schist' encountered in the excavations

for the Athens Metro and the graphitic phyllites encountered in some of the tunnels in Venezuela. This extension to GSI is based largely on the work of Maria Benissi on the Athens Metro.

- 2000 Hoek, E. and Marinos, P. (2000) Predicting Tunnel Squeezing. *Tunnels and Tunnelling International*. Part 1 - November Issue 2000,. 45-51, Part 2 - December, 2000, 34-36.
- 2000 Marinos, P.G. and Hoek, E. (2000): "GSI: A geological friendly tool for rock mass strength estimation", *Proceedings of the International Conference on Geotechnical & Geological Engineering (GeoEng 2000)*, Technomic Publishing Co. Inc., p.p. 1422-1440, Melbourne, Australia.
- 2001 Marinos. P, and Hoek, E. (2001) - Estimating the geotechnical properties of heterogeneous rock masses such as flysch, *Bull. Engg. Geol. Env.* 60, 85-92.

This group of papers puts more geology into the Hoek-Brown failure criterion than that which has been available previously. In particular, the properties of very weak rocks are addressed in detail for the first time. A new GSI chart for heterogeneous weak rock masses was introduced in these papers.

- 2002 Hoek, E., Carranza-Torres, C.T., and Corkum, B. (2002), Hoek-Brown failure criterion – 2002 edition. *Proc. North American Rock Mechanics Society meeting in Toronto in July 2002*.

This paper addresses the long running issue of the relationship between the Hoek-Brown and Mohr-Coulomb criteria. An “exact” method for calculating the cohesive strength and angle of friction is presented and appropriate stress ranges for tunnels and slopes are given. A rock mass damage criterion is introduced to account for the strength reduction due to stress relaxation and blast damage in slope stability and foundation problems. A Windows program called “RocLab” is developed to accompany this paper and is available for downloading (free) over the Internet.

APPENDIX A – DEVELOPMENT OF THE HOEK-BROWN CRITERION – SUMMARY OF EQUATIONS

Publication	Coverage	Equations
Hoek & Brown 1980	Original criterion for heavily jointed rock masses with no fines. Mohr envelope was obtained by statistical curve fitting to a number of (σ'_n, τ) pairs calculated by the method published by Balmer [1]. σ'_1, σ'_3 are major and minor effective principal stresses at failure, respectively σ'_t is the tensile strength of the rock mass m and s are material constants σ'_n, τ are effective normal and shear stresses, respectively.	$\sigma'_1 = \sigma'_3 + \sigma_{ci} \sqrt{m\sigma'_3/\sigma_{ci} + s}$ $\sigma'_t = \frac{\sigma_{ci}}{2} \left(m - \sqrt{m^2 + 4s} \right)$ $\tau = A \sigma_{ci} \left((\sigma'_n - \sigma'_t) / \sigma_{ci} \right)^B$ $\sigma'_n = \sigma'_3 + \left((\sigma'_1 - \sigma'_3) / (1 + \partial\sigma'_1/\partial\sigma'_3) \right)$ $\tau = (\sigma'_1 - \sigma'_3) \sqrt{\partial\sigma'_1/\partial\sigma'_3}$ $\partial\sigma'_1/\partial\sigma'_3 = m\sigma_{ci} / 2(\sigma'_1 - \sigma'_3)$
Hoek 1983	Original criterion for heavily jointed rock masses with no fines with a discussion on anisotropic failure and an exact solution for the Mohr envelope by Dr J.W. Bray.	$\sigma'_1 = \sigma'_3 + \sigma_{ci} \sqrt{m\sigma'_3/\sigma_{ci} + s}$ $\tau = \left(\text{Cot}\phi'_i - \text{Cos}\phi'_i \right) m\sigma_{ci} / 8$ $\phi'_i = \arctan \left(1 / \sqrt{4h \cos^2 \theta - 1} \right)$ $\theta = \left(90 + \arctan(1/\sqrt{h^3 - 1}) \right) / 3$ $h = 1 + \left(16(m\sigma'_n + s\sigma_{ci}) / (3m^2\sigma_{ci}) \right)$
Hoek & Brown 1988	As for Hoek (1983) but with the addition of relationships between constants m and s and a modified form of RMR (Bieniawski [2]) in which the Groundwater rating was assigned a fixed value of 10 and the Adjustment for Joint Orientation was set at 0. Also a distinction between <i>disturbed</i> and <i>undisturbed</i> rock masses was introduced together with means of estimating deformation modulus E (after Serafim and Pereira [3]).	<p><i>Disturbed rock masses:</i></p> $m_b/m_i = \exp((RMR - 100)/14)$ $s = \exp((RMR - 100)/6)$ <p><i>Undisturbed or interlocking rock masses</i></p> $m_b/m_i = \exp((RMR - 100)/28)$ $s = \exp((RMR - 100)/9)$ $E = 10^{((RMR - 100)/40)}$ <p>m_b, m_i are for broken and intact rock, respectively.</p>
Hoek, Wood & Shah 1992	Modified criterion to account for the fact the heavily jointed rock masses have zero tensile strength. Balmer's technique for calculating shear and normal stress pairs was utilised	$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left(m_b \sigma'_3 / \sigma_{ci} \right)^\alpha$ $\sigma'_n = \sigma'_3 + \left((\sigma'_1 - \sigma'_3) / (1 + \partial\sigma'_1/\partial\sigma'_3) \right)$ $\tau = (\sigma'_1 - \sigma'_3) \sqrt{\partial\sigma'_1/\partial\sigma'_3}$ $\partial\sigma'_1/\partial\sigma'_3 = 1 + \alpha m_b^\alpha \left(\sigma'_3 / \sigma_{ci} \right)^{\alpha-1}$
Hoek 1994 Hoek, Kaiser & Bawden 1995	Introduction of the Generalised Hoek-Brown criterion, incorporating both the original criterion for fair to very poor quality rock masses and the modified criterion for very poor quality rock masses with increasing fines content. The Geological Strength Index GSI was introduced to overcome the deficiencies in Bieniawski's RMR for very poor quality rock masses. The distinction between disturbed and undisturbed rock masses was dropped on the basis that disturbance is generally induced by engineering activities and should be allowed for by downgrading the value of GSI .	$\sigma'_1 = \sigma'_3 + \sigma_c \left(m\sigma'_3/\sigma_{ci} + s \right)^a$ <p>for $GSI > 25$</p> $m_b/m_i = \exp((GSI - 100) / 28)$ $s = \exp((GSI - 100) / 9)$ $a = 0.5$ <p>for $GSI < 25$</p> $s = 0$ $a = 0.65 - GSI/200$

Publication	Coverage	Equations
Hoek, Carranza-Torres, & Corkum 2002	An “exact” method for calculating the cohesive strength and angle of friction is presented and appropriate stress ranges for tunnels and slopes are given. A rock mass damage criterion is introduced to account for strength reduction due to stress relaxation and blast damage in slope stability and foundation problems. The “switch” at GSI = 25 for the coefficients s and a, is eliminated, which gives smooth continuous transitions for the entire range of GSI values. Corrections made to original Balmer equations.	$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left(m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^a$ $m_b = m_i \exp \left(\frac{GSI - 100}{28 - 14D} \right)$ $s = \exp \left(\frac{GSI - 100}{9 - 3D} \right)$ $a = \frac{1}{2} + \frac{1}{6} \left(e^{-GSI/15} - e^{-20/3} \right)$ $\sigma_c = \sigma_{ci} \cdot s^a$ $\sigma_t = -\frac{s \sigma_{ci}}{m_b}$ $\sigma'_n = \frac{\sigma'_1 + \sigma'_3}{2} - \frac{\sigma'_1 - \sigma'_3}{2} \cdot \frac{d\sigma'_1/d\sigma'_3 - 1}{d\sigma'_1/d\sigma'_3 + 1} \quad (\text{see note 1})$ $\tau = (\sigma'_1 - \sigma'_3) \frac{\sqrt{d\sigma'_1/d\sigma'_3}}{d\sigma'_1/d\sigma'_3 + 1} \quad (\text{see note 1})$ $d\sigma'_1/d\sigma'_3 = 1 + am_b \left(m_b \sigma'_3 / \sigma_{ci} + s \right)^{a-1} \quad (\text{see note 1})$ $E_m (GPa) = \left(1 - \frac{D}{2} \right) \sqrt{\frac{\sigma_{ci}}{100}} \cdot 10^{((GSI-10)/40)} \quad (\text{sigci} \leq 100)$ $E_m (GPa) = \left(1 - \frac{D}{2} \right) \cdot 10^{((GSI-10)/40)} \quad (\text{sigci} > 100)$ $\phi' = \sin^{-1} \left[\frac{6am_b (s + m_b \sigma'_{3n})^{a-1}}{2(1+a)(2+a) + 6am_b (s + m_b \sigma'_{3n})^{a-1}} \right]$ $c' = \frac{\sigma_{ci} \left[(1+2a)s + (1-a)m_b \sigma'_{3n} \right] (s + m_b \sigma'_{3n})^{a-1}}{(1+a)(2+a) \sqrt{1 + \left(6am_b (s + m_b \sigma'_{3n})^{a-1} \right)} / ((1+a)(2+a))}$ $\sigma_{3n} = \sigma'_{3\max} / \sigma_{ci}$

$$\sigma'_{cm} = \frac{2c' \cos \phi'}{1 - \sin \phi'}$$

$$\sigma'_{cm} = \sigma'_{ci} \cdot \frac{(m_b + 4s - a(m_b - 8s))(m_b/4 + s)^{a-1}}{2(1+a)(2+a)}$$

$$\frac{\sigma'_{3\max}}{\sigma'_{cm}} = 0.47 \left(\frac{\sigma'_{cm}}{\gamma H} \right)^{-0.94} \quad (\text{tunnels})$$

$$\frac{\sigma'_{3\max}}{\sigma'_{cm}} = 0.72 \left(\frac{\sigma'_{cm}}{\gamma H} \right)^{-0.91} \quad (\text{slopes})$$

NOTE 1: These are the corrected Balmer equations. The original equations were incorrect.

References

1. Balmer, G., 1952. A general analytical solution for Mohr's envelope. *Am. Soc. Test. Mat.* 52, 1260-1271.
2. Bieniawski, Z.T. 1976. Rock mass classification in rock engineering. In *Exploration for Rock Engineering, Proc. Of the Symp.*, (ed. Z.T. Bieniawski), 1, 97-106. Cape Town, Balkema.
3. Serafim, J.L. and Pereira, J.P. 1983. Consideration of the Geomechanics Classification of Bieniawski. *Proc. Intl. Symp. Engng. Geol. And Underground Construction*, Lisbon, Portugal, 1133-44.